Structural and Functional Image Registration Using Maximum Mutual Information

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Abstract

The technique of medical image radiography is developed for clinical diagnosis and application. Multi-modality image registration is designed to combine the information of medical image from different modalities and record at different times. The purpose is to register medical images such as magnetic resonance images (MRI) and positron emission tomography (PET) images using maximum mutual information (MI). The image registration system is designed as two phases: estimation of MI and image registration. One of the phases, MI is estimated the by statistical dependence or information redundancy between the image intensity of corresponding voxels in both images. The other phase is the parameters of the geometric transformation that are iteratively obtained by estimation of MI. In this paper, 2-D and 3-D registration techniques were developed and applied to medical image studies. The results show that it will be effective to use this study in image registration. The method is both accomplishable and suitable for clinical application.

Keywords: Medical image registration, Mutual information, Iterative process

Introduction

The medical images from different modalities such as magnetic resonance images (MRI) and positron emission tomography (PET) images are applied to clinical diagnosis and application. The information of medical image is useful to medical treatment and surgery. Based on image functions, medical images can be divided into two classes. One is the structural emission image can provide high-resolution tissue structure such as MRI, X-ray image and CT. The other is functional tomography image can provide physiological information of tissue like PET images and functional Magnetic Resonance Image (fMRI).

Multi-modality image registration is the basic task in three-dimensional (3-D) and has numerous applications. In registration the main problem is the choice of parametric transformations and the estimation of its parameter after extracting and pairing common features. Image registration by maximum MI has been presented whereas MI has been applied to measure the amount of information between two images. The information such as image intensity, histogram, the shape or gradient of image and parameterization of the spatial mapping features such as location, indicates any common feature is extracted from two objects. MI is usually derived from joint probability or entropy of the feature and often interpreted as a measure of uncertainty, variability, or complexity. Relying to the information correlation, MI that can be successfully employed in feature-based image registration is a surprising fact.

Recently, using MI-based similarity measures in voxel-based (VB) registration has been presented [1][2]. VB registration algorithms optimize a function that measures the similarity of all possible pairs in all geometrically corresponding voxel pairs between two images. The main advantage of VB method is that features are computed directly, only gray-values are used; furthermore, it will not be limited by segmentation errors. Feature location information for image registration has been presented [3]. The method is just like surface-based registration which finding shape or surface of the images are needed. Surfaced segmentation algorithms are generally both high data and application dependent and surfaces are hardly identified in functional modalities such as PET. A gradient-descent approach has been shown [4] which computes a stochastic approximation for the gradient of the MI criterion from an estimation of the joint histogram derived by Parzen windowing from a very limit number of samples.

In this paper, the registration system is designed based on the VB method which determines the best registration by maximizing the similarity between images of the same object without segmentation and extract processing, thus it can

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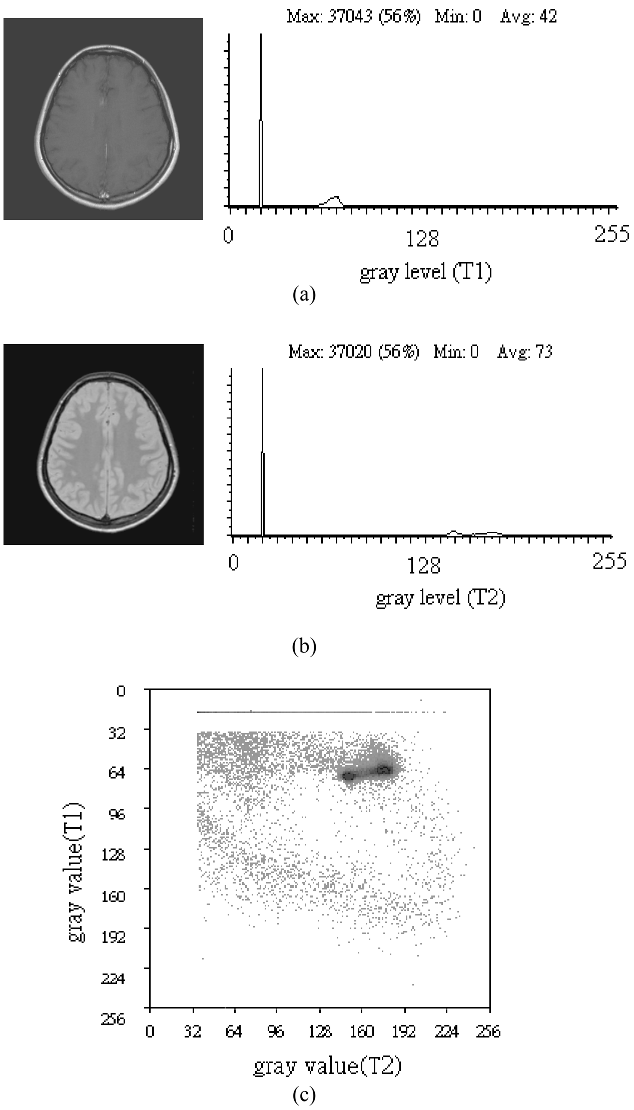


Figure 1. Structural medical images, (a) T1-weighted MRI (Brightness: 45%) and histogram, (b) T2-weighted MRI and histogram, (c) the joint histogram of T1- and T2-weighted MRI.

prevent original information of images from image distortion. The estimation of MI is used [1-2]. The registration system is designed from the parameters of geometric transformation iteratively obtained by MI. Advantages of MI registration criterion are no limiting constraints on the nature of the relation between the intensities of two images to be registered, no assumptions are made regarding to image content of the modalities. It allows for robust and complete automated registration of multi-modal images without prior segmentation, feature extraction or other preprocessing steps.

This paper is organized as follows. The materials for performing registration system, the theory and estimation of MI and the implement of the registration system, in which parameters of geometric transformation are iteratively obtained by MI and are described in section 2. Section 3 shows the accuracy of the registration using MI for phantoms and medical images including MRI and PET image in 2D and 3D, while section 4 discusses our current findings and conclusions.

Materials and Methods

Data Acquisition

All the MRI studies were acquired by a 1.5-telsa clinical MRI system (Magnetom Vision, Siemens, Germany) using a standard circularly polarize head coil. The acquisition parameters for T1-weighted MRI studies are TR/TE = 400/12 ms, slice-thickness = 6 mm, matrix = 256 x 256, pixel size = 0.84 x 0.84 mm and scan time = 2 min 18 sec and T2-weighted MRI studies are TR/TE = 5000/20 ms, slice-thickness = 3 mm, matrix = 256 x 256, pixel size = 1 x 1 mm and scan time = 12 min 51 sec. All the PET images studies were acquired by GE/Scanditronix PET camera (PC4096-15WB). Fifteen emission frames were acquired immediately after IV injection of 5-10 mCi of F-18 FDG. Fifteen emission frames were acquired immediately after iv injection of 50-60 mCi of H_2O^{15}. The acquisition parameters are slice-thickness = 6.5 mm, matrix = 128 x 128, pixel size = 2 x 2 mm.

Mutual Information

Suppose U and V be two random variables, $I(U,V)$ is MI of the partition U and V, the reduction in the uncertainty of the random variable U by the knowledge of another random variable V [2][5-6]. On the other hand, $I(U,V)$ can be interpreted as the "information about U contained in V " and it equals the " information about V contained in U ". u and v are a pair of feature of two objects, which marginal probability distributions is $p(u)$, $p(v)$ and joint probability distribution is $p(u,v)$. $I(U,V)$ measures the degree of dependent of U and V by measuring the Kullback-Leibler distance [7-8] between the joint distribution $p_{UV}(u,v)$ and the distribution to the case of complete independent $p_U(u) . p_V(v)$:

$$I(U,V) = \sum_{u,v} p_{UV}(u,v) \log \frac{p_{UV}(u,v)}{p_U(u).p_V(v)} \tag{1}$$

When U and V are statistically independent, $p_{UV}(u,v) = p_U(u).p_V(v)$ and $I(U,V) = 0$.

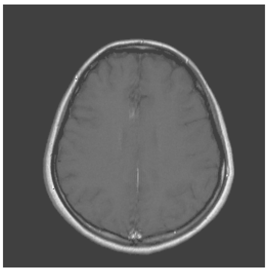
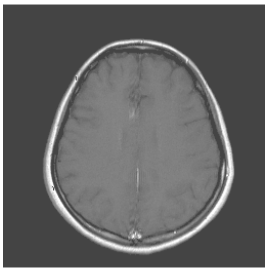
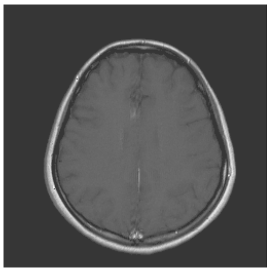
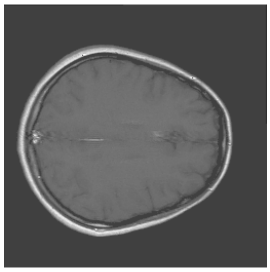
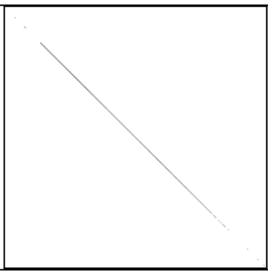
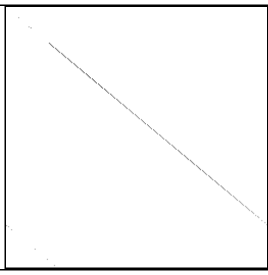
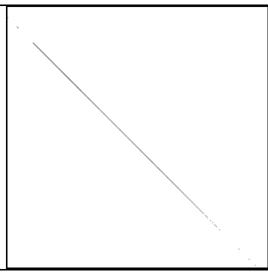
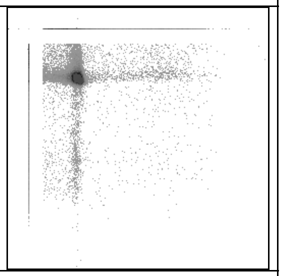
MI has been described to measure the information or feature between two images. Equation (2) is developed from equation (1) [9] that image intensity statistics, histogram is used to estimate MI between two images.

$$I(U,V) = \sum_u \sum_v \frac{h(u,v)}{N} \log \frac{\frac{h(u,v)}{N}}{\frac{h(u)}{N} * \frac{h(v)}{N}} \tag{2}$$

U and V are two images. u and v are a pair of image intensity of two images. $h(u,v)$ is joint histogram of u and v, and $h(u)$, $h(v)$ are histogram of u and v respectively. N is the number of u and v.

MI is computed by medical images actually which were T1- and T2-weighted MRI and individual histogram shown in figure 1 (a) and (b). Both pixel values of corresponding location images are used to estimate the joint histogram computed from T1-weighted MRI and T2-weighted MRI as shown in figure 1(c). According to the statistics of individual

Table 1. The similarity of MI and linear and nonlinear changing the pixel value of medical images, T1-weighted MRI (Brightness: 45%).

	$I(T1,T1)$	$I(T1,1.2T1)$	$I(T1,T1-10)$	$I(T1,T1')$
Floating image				
Joint histogram				
MI	2.3214	2.3214	2.3214	0.4243
Similarity of MI	100%	100%	100%	18.28%

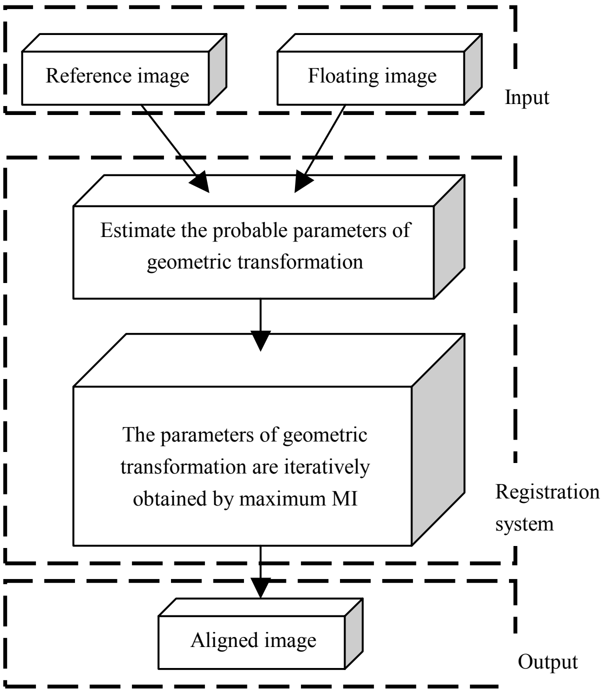


Figure 2.System block diagram of the MI-based medical image registration.

histogram and joint histogram, MI can be computed by equation (2). Estimation of MI, $I(T1,T2)$ estimated by medical images, is 0.9793 .

The similarity of MI is defined by normalizing estimation of MI from one of the two images and identity as described in equation (3).

$$\textit{Similarity of MI} = \frac{I(U,V)}{I(U,U)} \times 100\% \tag{3}$$

The maximum similarity of two images is 100%. By estimating the larger similarity of MI, the two images are more similar. On the contract, the two images are non-homologies.

The similarity of MI is obtained by changing the image intensity of medical images, T2 to 1.2T1, T1-10 and T1'. 1.2T1 is the pixel values of T1 multiplied by 1.2, T1-10 is reducing the pixel values of T1 by 10, and T1' is T1 rotated clockwise by 90 degree. The images above, the joint histogram, the estimation and similarity of MI are shown in table 1. When images are similar, the joint histogram came together in diagonal direction. By changing image intensities linear, the joint histogram still came together in diagonal direction with a weakly shifting but the estimation and similarity of MI are the same with the original image. The image intensities are changed nonlinearly and the joint histogram will spread based on the location of image intensities. MI of the image intensity values of corresponding voxel pair is maximum means the similarity between the images is largest.

Registration

Registration as geometric transformations has been defined as the determination of a one-to-one mapping between the coordinates in one space and those in another such that points in two spaces that correspond to the same anatomical point are mapped to each other [1]. Mapping is also called transformation, which are two-dimensional to two-dimensional space and three-dimensional to three-dimensional space. Geometric transformations include Pixel co-ordinate transformation and Brightness interpolation [10].

In this paper, the MI registration criterion states that the images are geometrically aligned by the transformation in which MI is maximal. The structure of the block diagram shown in figure 2 describes the MI registration system. One of the images is selected to be floating image, F, then registered to reference image, R, and both images are the inputs. The

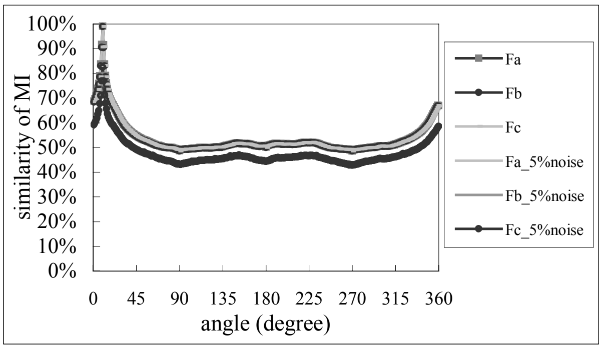
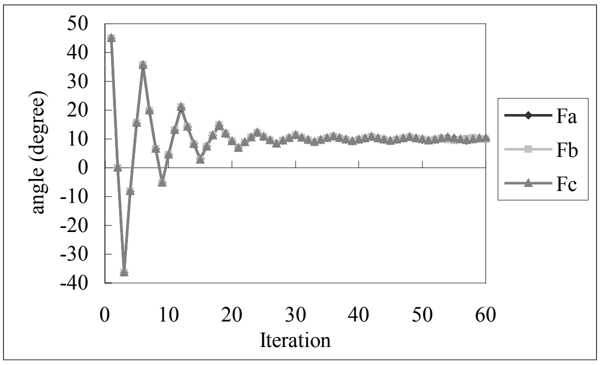
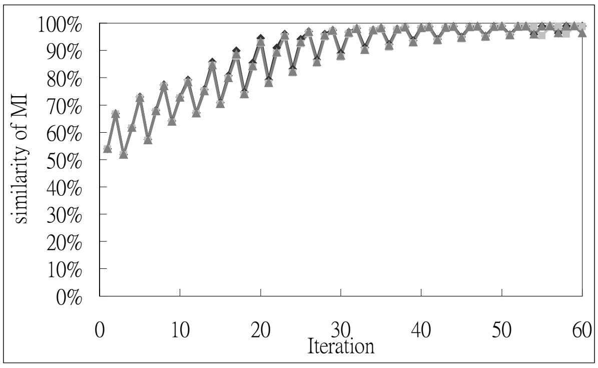


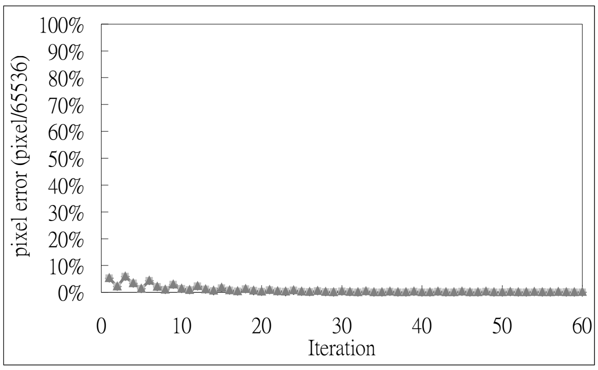
Figure 3. The maximum similarity of MI presents the optimal aligned result of two images.



(a)



(b)



(c)

Figure 4. The count of iteration is set as 60, and (a) angle, (b) the variation in similarity of MI and (c) pixel error are going to stable.

Table 2. The aligned mean error and standard deviation of angles and displacements for 2D simulation.

error / (R,F)	$\Delta\theta_z$ (degree)	Δt_x (pixel)	Δt_y (pixel)
(F_1, R)	0.0436 ± 0.0414	0.0444 ± 0.0318	0.0496 ± 0.036
(F_2, R)	0.0579 ± 0.0495	0.0514 ± 0.0356	0.0410 ± 0.0277
(F_3, R)	0.0635 ± 0.047	0.0517 ± 0.0417	0.0542 ± 0.0436

aligned image is the output of registration system. The registration system is based on probable parameters of geometric transformation estimated from previous registration method or any initialized random variable and then the parameters of geometric transformation are iteratively obtained by estimation of MI and the iterative function as in equation (4) [11].

$$\phi^{n+1} = \phi^n \pm w^* \cdot \Delta\phi \tag{4}$$

ϕ^n is the parameters of geometric transformation at iterative count n. The angle error $\Delta\phi$ is differed between ϕ^n and ϕ^{n-1} which has both the next adjust magnitude and direction. If the weighting value is greater than 1, the direction will be opposite. On the contrary the weighting value is less than 1, and the direction will be changeless. w^* is a weight function based on MI and n is the iterative count.

The weighting value in equation 5 is ratio by the MI between the position at iterative count n and at iterative count n-1.

$$w^* = \begin{cases} \frac{1}{w}, & if\ I(R,F(\phi^n)) \geq I(R,F(\phi^{n-1})) \\ w, & otherwise \end{cases}, \qquad w = \frac{I(R,F(\phi^{n-1}))}{I(R,F(\phi^n))} \tag{5}$$

The parameters of the geometric transformation and similarity of MI obtained from equation 4 is going to be stable by several iterations. When $I(R,F(\phi))$ is maximal and reference and floating images are geometric aligned [2][4].

$$I(R,F(\phi)) = \sum_{f,r} h_{FR}(r,f) \log \frac{h_{FR}(r,f)}{h_F(r) \cdot h_R(f)} \tag{6}$$

$$\phi^* = \arg\max_{\phi} I(R,F(\phi)) \tag{7}$$

$I(R,F(\phi))$ is the mutual information of R and F at position ϕ, and ϕ^* is the position at which $I(R,F(\phi))$ is maximal.

The main goal of registration system is to without image preprocessing. In the previous and rear estimating, the parameters of geometric transformation and MI weighting function are used to design an iterative registration system. With the maximum similarity of MI, the reference and floating images are geometric aligned.

Table 3. The aligned mean error and standard deviation of angles and displacements for 2D simulated images with different noise level.

(R,F) \ error	$\Delta\theta_z$ (degree)	Δt_x (pixel)	Δt_y (pixel)
(F_1_5%noise, R)	0.1332 ± 0.0333	0.0954 ± 0.0594	0.0559 ± 0.045
(F_2_5%noise, R)	0.1465 ± 0.0401	0.1106 ± 0.0764	0.1290 ± 0.0775
(F_3_5%noise, R)	0.1295 ± 0.0545	0.0715 ± 0.0473	0.0792 ± 0.0671
(F_1_10%noise, R)	0.1779 ± 0.1393	0.1364 ± 0.1078	0.1098 ± 0.0759
(F_2_10%noise, R)	0.1527 ± 0.0392	0.1250 ± 0.1409	0.1236 ± 0.0881
(F_3_10%noise, R)	0.1473 ± 0.0216	0.0932 ± 0.0641	0.1260 ± 0.0867

Table 4. Results of 2D simulated studies, for different image modalities and different radiopharmaceutical tracers.

(R,F) \ error		$\Delta\theta_z$ (degree)	Δt_x (pixel)	Δt_y (pixel)
(MRI, MRI)	(T1, T1)	0.0261 ± 0.0167	0.0168 ± 0.0125	0.0139 ± 0.0111
	(T2, T2)	0.0183 ± 0.0132	0.0118 ± 0.0104	0.0101 ± 0.0065
	(T1, T2)	0.2423 ± 0.1812	0.3058 ± 0.2024	0.1806 ± 0.1055
(PET, PET)	(FDG, FDG)	0.0095 ± 0.0086	0.0112 ± 0.0079	0.0080 ± 0.0067
	(H_2O^{15}, H_2O^{15})	0.0103 ± 0.0073	0.0122 ± 0.0083	0.0062 ± 0.0055
	(FDG, H_2O^{15})	0.3614 ± 0.267	0.0990 ± 0.0882	0.1660 ± 0.1155
(MRI, PET)	(T1, FDG)	0.4202 ± 0.3101	0.5275 ± 0.3918	0.3718 ± 0.2477
	(T1,W)	0.6567 ± 0.4329	0.4742 ± 0.318	0.5033 ± 0.4104

Table 5. Mean results of 3D phantom.

(R,F) \ error	$\Delta\theta_x$ (degree)	$\Delta\theta_y$ (degree)	$\Delta\theta_z$ (degree)	Δt_x (pixel)	Δt_y (pixel)	Δt_z (pixel)
(Phantom, Phantom)	0.3383 ± 0.5384	0.3684 ± 0.3533	0.4368 ± 0.6127	0.0699 ± 0.0736	0.1093 ± 0.2259	0.1412 ± 0.4656

Experiments and Results

In this section, the registration experiments and results are obtained by the registration system designed from mutual information criterion for various applications. A 2-D phantom is simulated to present the properties of MI based on the method we approached. This method is used to test image registration system by modifying phantom image intensities, linear and nonlinear and adding different ratio random white noise level. The registration experiments of multi-modality medical images such as 2-D and 3-D MRI and PET images were accomplishable. The parameters of geometric transformation in 2-D space include one rotation angles and two translation distances. 3-D space contains three rotation angles and three translation distances.

2-D Phantom

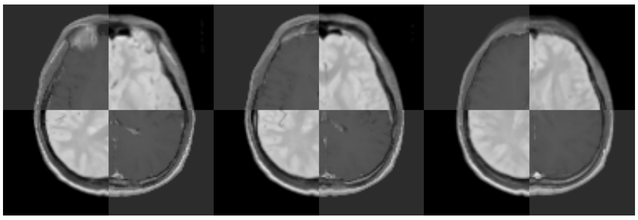
The 2-D phantom obtained from the segmentation of brain tissues such as gray and white matter in T2-weighted MRI. The properties of MI are implemented when the original phantom as reference image is rotated counterclockwise by 10 degrees and image intensities linear changed or/and added 5% random white noise to be the floating images. Fa can be generated by rotating the original phantom counterclockwise for 10 degrees in X-Y plane. Fb is generated by adding ±13 and ±5 to the gray and white matter of Fa. Fc is generated by multiplying 1.3 and 0.9 to the gray and white matter of Fa. By adding 5% random white noise to Fa, Fb and Fc, Fa_5%noise, Fb_5%noise and Fc_5%noise can be obtained. By rotating the float images above from 1 to 360 degrees to match original phantom, similarity of MI computed are shown in figure 3. Each maximum MI is at the same angle, 10 degrees. According to the result of compute and variation, MI can clearly and easily point out the rotated angle of reference and floating image, which is the closest to actual condition even when image intensity is linear or has been changed even add noise.

Using floating images above, Fa, Fb and Fc, to perform the registration system in which the parameters of geometric transformation are iteratively obtained by MI estimation. The count of iteration is set as 60. The variation in similarity of MI, angle and pixel error are shown in figure 4. At the beginning, At beginning the variation in similarity of MI, angle and pixel error, are more serious. Similarity of MI becomes larger and larger means that the reference and floating images are more similar. Pixel error becomes smaller means that the correlation of reference and floating images is positive. The angles shock up and down. After several iterative estimations, MI similarity, angle, and pixel error are going to stable. The aligned data (Fa, Fb, Fc) are chosen with maximum MI similarity (98.96%, 98.71%, 98.8%). Iterations are (56, 57, 59). Angle errors are (0.1534 degree, 0.1474 degree, 0.1696 degree) and pixel errors (pixel/65536) are (0.0092%, 0.0137%, 0.0153%).

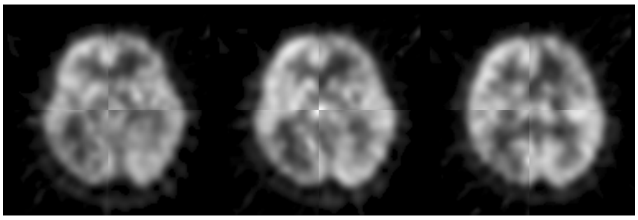
The twenty floating images, F_1 are obtained from original phantom random transformed by the parameters of geometric transformation which angle (θ_x) range is –30.0~+30.0 degrees and the displacement (t_x, t_y) are +5.0~-5.0. The image intensities are linear changed, which F_2 is generated by

Table 6. The aligned mean error and standard deviation of angles and displacements for 3D images registration.

error / (R,F)	$\Delta\theta_x$ (degree)	$\Delta\theta_y$ (degree)	$\Delta\theta_z$ (degree)	Δt_x (pixel)	Δt_y (pixel)	Δt_z (pixel)
(T1,T1)	0.2787 ± 0.5531	0.1852 ± 0.1299	0.4111 ± 0.6697	0.0640 ± 0.0715	0.1186 ± 0.2232	0.2154 ± 0.512
(T2,T2)	0.3051 ± 0.5619	0.1680 ± 0.1641	0.3455 ± 0.6963	0.0866 ± 0.0843	0.1165 ± 0.2256	0.2431 ± 0.4539
(FDG, FDG)	0.2896 ± 0.5461	0.1990 ± 0.1826	0.3352 ± 0.678	0.0635 ± 0.0723	0.1126 ± 0.2283	*0.2207 ± 0.4402*
(H_2O^{15}, H_2O^{15})	0.2823 ± 0.5519	0.2073 ± 0.1416	0.3044 ± 0.652	0.0646 ± 0.0735	0.1111 ± 0.2196	0.2375 ± 0.4604



(a)



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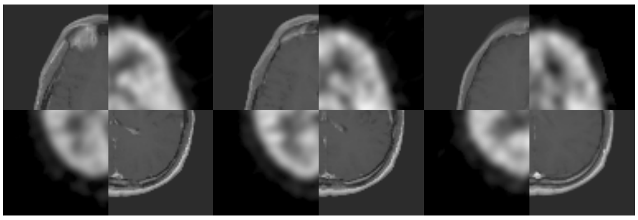

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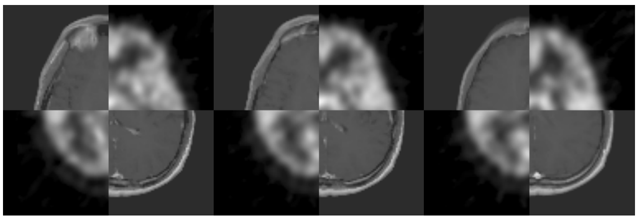

(d)

Figure 5. The aligned result of (a) T2-weighted MRI registered to T1-weighted MRI (Brightness: 45%), (b) H_2O^{15}-PET images registered to FDG-PET images and (c) H_2O^{15}- and (d) FDG-PET images registered to T1-weighted MRI (Brightness: 45%) are three slices of images ($5^{th}/26$, $10^{th}/26$, $17^{th}/26$). The upper left and lower right side is reference image and otherwise is the aligned result.

adding ± 13 and ± 5 and F_3 is generated by multiplying 1.3 and 0.9 to the gray and white matter of F_1. The aligned mean error, standard deviation of angles and displacements are showed in table 2. The mean error of angles is less than 0.07 degrees and displacements in X and Y plane is less than 0.06 pixels.

The twenty floating images, which image intensities are linear changed and added different random white noise level twenty times, are obtained from original phantom. The original phantom is transformed by the parameters of geometric transformation, which the angle (θ_x) is 15 degrees clockwise and the displacement (t_x, t_y) is (6,6). F_1_5%noise, F_2_5%noise, F_3_5%noise, F_1_10%noise, F_2_10%noise and F_3_10%noise are generated by adding 5% and 10% random white noise to F_1, F_2 and F_3. The aligned mean error and standard deviation of angles and displacements are shown in table 3. The mean error of angles is less than 0.2 degrees and displacements in X and Y plane is less than 0.2 pixels while adding 10% random white noise to floating images.

2-D medical images

The medical images such as structural images, T1-, T2-weighted MRI and functional images, FDG-PET and H_2O^{15}-PET images are used to perform our registration system. The medical images above are randomly transformed to be the twenty floating images registered to original image by the parameters of geometric transformation. The angle (θ_x) range is +30.0~-30.0 degrees. The displacement (t_x, t_y) is +5.0~-5.0. The aligned data MRI – MRI, PET images – PET images and MRI – PET images are presented in table 4 which shows the accuracy of registration. The mean error of angles is less than 0.03 degrees and displacements in X and Y plane is less than 0.02 pixels while the reference images and floating images are in the same modality. The mean error of angles is less than 0.7 degrees and displacements in X and Y plane are less than 0.6 pixels while the reference images and floating images are multi-modality. Register identical image is more accurate than different images.

3-D phantom

A 3-D phantom is a PET image with seven different volumes which are injected into the same radiopharmaceutical. The experiment is practiced by several slices extracted form 3-D phantom. The slices are randomly transformed into twenty floating image by the parameters of geometric transformation. The angle $(\theta_x, \theta_y, \theta_z)$ range is +5.0~-5.0 degrees, the displacement (t_x, t_y) are +5.0~-5.0 and (t_z) is +2.5~-2.5 slice. Registering the floating images to original images shows 3-D registration performance. The aligned mean error, standard deviation of angles and displacements are shown in table 5. The mean error of angles is less than 0.5 degrees, displacements in X and Y plane is less than 0.2 pixels and in Z direction is less than 0.2 slices.

Volume of MRI and PET images

The experiment is designed as two parts. One is to transform the original images such as structural images, T1- and T2-weighted MRI and functional images, FDG-PET and H_2O^{15}-PET images randomly in 3-D space by the parameters of geometric transformation. The angle, displacement and slice range are the same with 3-D phantom experiment above. By registration processing can shows the performance of registering volume MRI and PET images, the aligned mean error, standard deviation of angles and displacements are shown in table 6. The mean error of angles is less than 0.5 degree, displacements in X and Y plane is less than 0.2 pixels and in Z direction is less than 0.25 slices.

The other is to register the medical images above with mutual information criterion for 3-D volume of MRI and PET brain images of the same normal person but recorded at different times. The aligned result of the same modalities, T2-weighted MRI registered to T1-weighted MRI, H_2O^{15}-PET images registered to FDG-PET images and multi- modalities, H_2O^{15}- and FDG-PET images registered to T1-weighted MRI are three slices of images ($5^{th}/26$, $10^{th}/26$, $17^{th}/26$). They are shown in figure 5 in which the upper left and lower right side is reference image while otherwise is the aligned result. Aligned result observation of both brain tissue and skull, the 3-D volume of medical images are geometric aligned.

Conclusion

The methods to perform registration have been increasingly automatic, efficient, and robust. Medical image registration is designed to combine the information of medical images from different modalities and record it at different times. The purpose is to register medical images such as MRI and PET images with maximum MI. The image registration system is tested and verified by experiments above. Image registration can align multi-modality medical images such as 2-D and 3-D successfully.

MI matching criterion allows us to have accurate, robust and completely automated registration of multi-modality medical image. The major benefit of this method is to register images without segmentation and preprocessing. Further research is tune to have the method optimized and extending the MI theory to image deformation.

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