Quantitative Analysis of Noise Influence on the Detection of Scatterer Concentration by Nakagami Parameter

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Abstract

The Nakagami statistical parameter has been demonstrated to be capable of detecting the variation of scatterer concentration in a biological tissue. The accuracy and sensitivity of the Nakagami parameter would be varied by the quality of ultrasonic backscattered signals, which is further affected by noise interference. For this reason, the experimental measurements and two-dimensional computer simulations were carried out to explore the effect of noise on the estimation of the Nakagami parameter as a function of scatterer concentration. The noise of the practical measurement system was acquired and subsequently its probability density function (PDF) and the Nakagami parameter were calculated. The incorporated simulation study was performed to evaluate the performances of the Nakagami parameter estimated from ultrasonic backscattered signals associated with different levels of signal-to-noise ratio (SNR) by adding simulated white noises with 5 MHz ultrasonic echoes corresponding to different scatterer concentrations. The obtained results showed that the envelope of white noise follows the Rayleigh distribution discernible by the calculated Nakagami parameter close to unity. Moreover, the sensitivity of Nakagami parameter to differentiate different scatterer concentrations decreased gradually corresponding to the decrease of SNR of backscattered signals. The SNR of backscattered signals was further suggested to be at least higher than 11 dB to affirm a satisfactory performance of the Nakagami parameter for characterizing the properties of biological tissues.

Keywords: Ultrasonic tissue characterization, Nakagami parameter, White noise

Introduction

The Nakagami statistical model initially proposed to describe the statistics of returned radar echoes has been extensively applied to the ultrasound characterization of tissue [1]. Compared to other statistical models, this model is relatively general and simple for practical applications. Two of its associated parameters, the Nakagami parameter m and the scaling parameter O, have been demonstrated to be capable of quantitating the scatterer concentration in biological tissues [1]. The Nakagami and scaling parameters respectively can be calculated using

$$m = \frac{[E(R^2)]^2}{E[R^2 - E(R^2)]^2}, \quad (1)$$

and

$$\Omega = E(R^2), \quad (2)$$

where E(•) is the statistical mean and R represents the envelope of ultrasonic backscattered signals. The Nakagami parameter in particular is feasible to characterize the probability density function (PDF) of ultrasonic backscattered envelope, including the statistical conditions for pre-Rayleigh, Rayleigh, and post-Rayleigh distributions. Values of m ranging from 0 to 1 reflect various PDFs from pre-Rayleigh to Rayleigh distributions and those of higher than 1 correspond to the PDFs of post-Rayleigh or Rician distributions. Due to various arrangements of scatterers in a tissue may result different backscattered statistics, the Nakagami parameter was further applied to classify the properties of tissues [2,3]. For better quantitating tissues using the Nakagami parameter, some factors could affect its estimation, such as pulse length, beam width, and attenuation, were taken into account [4,5]. In addition, the quality of ultrasonic backscattered signals, which would be affected by noise contamination in the measurements, might also be another key factor to influence the accuracy and sensitivity of the Nakagami parameter, especially in high frequency applications.

In general, electrical noises according to their sources can be classified into external and internal noises. The sources of external noise are numerous and complex, which mainly originate from the environment and user operation such as electromagnetic fields, instrument switching, or personnel

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contacting [6]. Internal noises on the other hand stem from the electronic components and conduction wires within a system that may be a summation of various noises including thermal noise, shot noise, flicker noise, partition noise, and burst noise [7]. Due to those external and internal noises respectively exhibit different frequency characteristics, the white noise, the summation of noises from all aspects, can be used to broadly evaluate the noise issue. White noise is the most commonly encountered electrical noise [8] that affects the characteristics of input and output signals. Therefore, waveforms of the ultrasonic backscattered envelope varied with the decrease of signal-to-noise ratio (SNR) of backscattered signals may readily lead to variations in the PDF and associated errors for the subsequent estimation of the Nakagami parameter. Certainly, the presence of noise will reduce the accuracy and sensitivity of the Nakagami parameter in detecting the variation of scatterer concentration.

Accordingly, this study is to carry out both the experimental measurements and two-dimensional computer simulations to further systematically comprehend the effect of noise on the performance of the Nakagami parameter. The statistical distribution and corresponding Nakagami parameter calculated from the acquired white noises were analyzed. The variation of the Nakagami parameter associated with scatterer concentrations of different SNRs was further explored. The results were discussed thoroughly to better elucidate the relation between the SNR of ultrasonic backscattered signals and the sensitivity of the estimated Nakagami parameter for ultrasound tissue characterization.

Materials and methods

The white noise of the measurement system was acquired and its corresponding PDF and Nakagami parameter were calculated. The whole measurement system consists of a pulser/receiver (Model PR5072, Panametrics, Waltham, MA) and a 500 MHz A/D board (PDA-500, Signatec, Corona, CA) inserted in a personal computer (Pentium Ⅲ-667 MHz). The pulser/receiver equipped with a built-in amplifier of a bandwidth of 35 MHz may be operated at two modes in which mode 1 and 2 are respectively for pulse-echo and transmission measurements. The white noise generated from the whole measurement system under normal operation at mode 2 without connecting to a transducer was acquired. Therefore, the white noise corresponding to any possible interference with the frequency response of the ultrasonic transducer was excluded. In each experiment, a total of 256 gated white noises, in which each unfiltered signal comprises of a 1-cm-long window amplified by a 50 dB, were acquired at a sampling rate of 50 MHz. The envelope of each white noise calculated using the Hilbert transform was then applied to estimate the PDF and the Nakagami parameter. The experiment was repeated for five times and the mean and standard deviation of the results were calculated.

For the corresponding computer simulations, a system-based model [9,10] was used to calculate backscattered signals from scatterers of various concentrations. Assuming weak scatterers are located in the range cell, the Born approximation was applied to calculate backscattered signals by summing echoes contributed from each individual scatterer using the superposition principle. Therefore, the received backscattered signals can be modeled as

$$RF_{3d}(x,y,z)=\frac{\partial^2}{\partial y^2}T_{3d}(x,y,z)\otimes Z_{3d}(x,y,z), \quad (3)$$

where $T_{3d}(x,y,z)$ represents the impulse response of the transducer and $Z_{3d}(x,y,z)$ is the acoustic impedance of the scatterers, which can be further formulated as

$$Z_{3d}(x,y,z)=C_{3d}(x,y,z)\otimes N_{3d}(x,y,z), \quad (4)$$

in which $C_{3d}(x,y,z)$ denotes the scatterer prototype corresponding to the size and shape of scatterers, and $N_{3d}(x,y,z)$ is the spatial distribution function of scatterers given as follows.

$$N_{3d}(x,y,z)=\sum_{n=1}^{M}d(x-x_n,y-y_n,z-z_n). \quad (5)$$

$N_{3d}(x,y,z)$ can also be interpreted as the number of scatterers, M, in each sample volume. Consequently, the backscattered signals can be computed by convolving the transducer transfer function with both the scatterer prototype and the spatial distribution of scatterers.

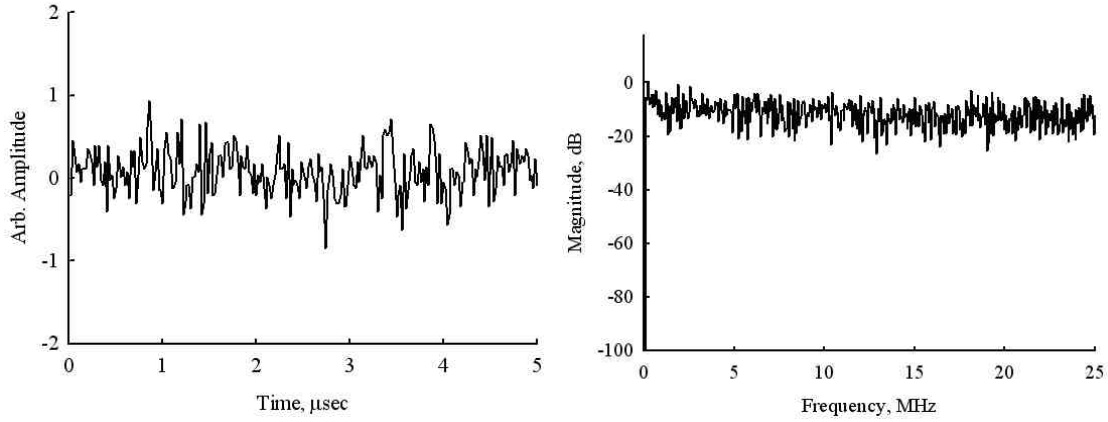
Instead of 3-D analysis, the computer simulations were performed using the 2-D model to alleviate computational complication and computer storage. Some assumptions including the attenuation of tissues and the diffraction of the transducer were also ignored. Moreover, those scatterers were considered to distribute randomly in the water and are with circular shape and the sound velocity of 1540 m/s. The incident pulse of ultrasound was assumed to be the Gaussian shape with the far-field power density pattern estimated as [11]

$$H(x)=\left(\frac{2J_1(kax/z)}{kax/z}\right)^2, \quad (6)$$

where J_1 is the Bessel function of the first kind with the order of 1, a and z represent the radius and focal length of the transducer, respectively, and k is the wave number. In addition, neglecting the effect of the scatterer prototype and considering the existence of noise, the final ultrasonic backscattering model used for simulation was simplified to be

$$RF_{3d}(x,y,z)=\frac{\partial^2}{\partial y^2}T_{3d}(x,y,z)\otimes N_{3d}(x,y,z)+n_{3d}(x,y,z). \quad (7)$$

in which $n_{3d}(x,y,z)$ is signal independent Gaussian white noise of zero mean [12]. The simulation program was developed using MATLAB (Version 6, MathWorks Inc., Natick, MA) on a personal computer. Phantoms for computer simulation containing scatterers at concentrations ranged from 2 to 32 scatterers/mm^2 were constructed. The positions of scatterers in each phantom were determined using the results generated



(a) (b)

Figure 1. A typical white-noise signal (a) and its spectrum (b) in the experiments.

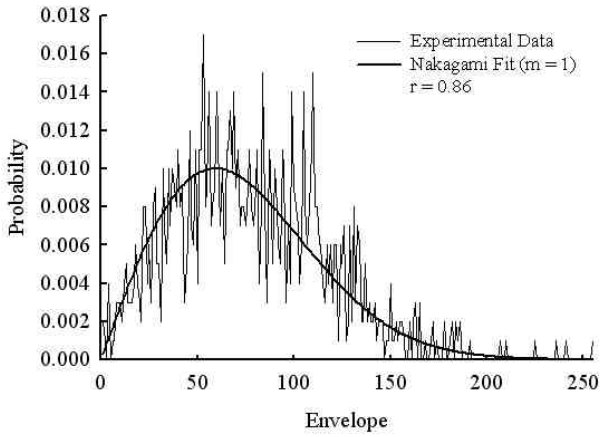
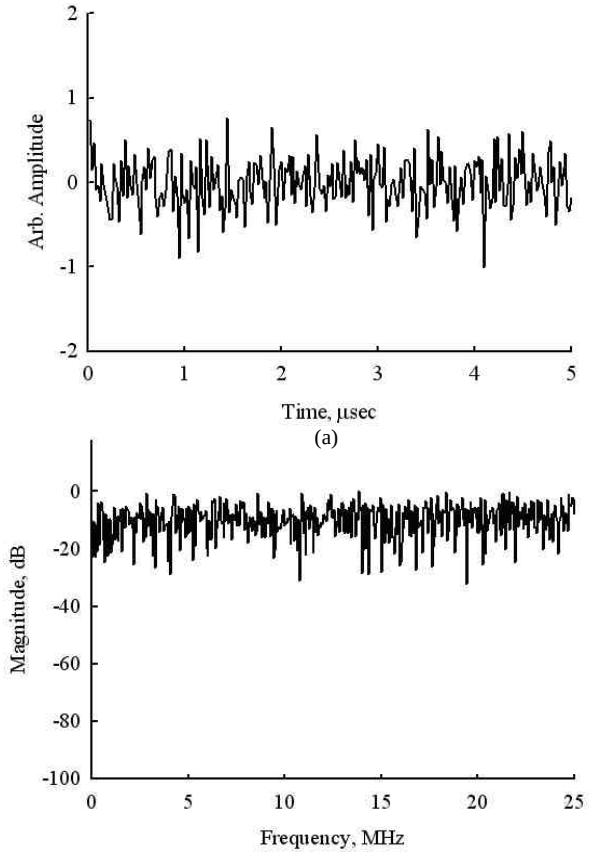


Figure 2. The PDF and curve fitting using the Nakagami parameter for the white noise envelope obtained in the experiments.

from a normal stochastic process followed by the central limit theorem [9]. The incident ultrasound of a 5 MHz central frequency was assumed to have a full-width half-maximum bandwidth of 3 MHz and a –3 dB beamwidth of 0.5 mm. The received backscattered signals were acquired from the 2-D convolution of the incident wave with the simulated phantom. Totally, for a specific scatterer concentration in the phantom, five simulations were performed. For each simulation, 256 A-lines of backscattered signals digitalized at a 50 MHz sampling frequency were acquired, in which each A-line corresponds to a gated window of 1-cm-long. To further explore the effect of white noise on the estimation of the Nakagami parameter, each simulated backscattered signal was added with Gaussian white noises of different levels to implement SNRs covering values of infinite, 40, 20, 10, 5, and 0 dB. Finally, the envelopes of backscattered signals of various SNRs were calculated by the Hilbert transform and which are for the subsequent calculation of PDF and the Nakagami parameter.

Results

A typical waveform of white noise and the corresponding spectrum obtained from the measurement system are shown in Figure 1. The acquired white noise behaved as a random signal



(a)

(b)

Figure 3. A typical white-noise signal (a) and its spectrum (b) in the simulations.

with a spectrum density approximately equal to uniform distribution in spite of the limited bandwidth of noise by the frequency response of the measurement system. Figure 2 is the experimental results of a typical PDF of the envelope of white noise and the corresponding curve fitting by the Nakagami model, in which the average Nakagami parameter is 1.01 ± 0.02 and the correlation coefficient between the experimental data and Nakagami PDF is 0.86. It indicates that the envelope of white noise follows the Rayleigh distribution. The simulated white noise and its corresponding spectrum are shown in Figure 3. The PDF for the envelope of simulated

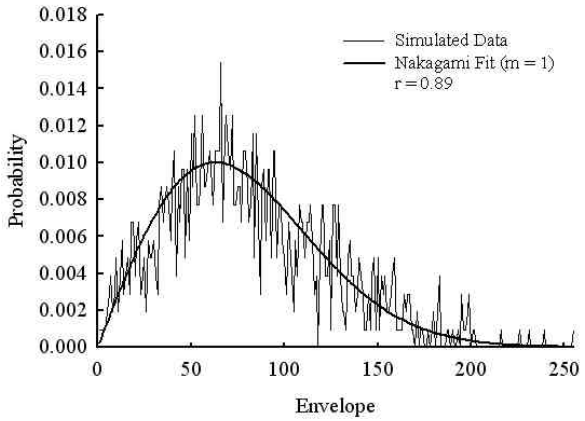


Figure 4. The PDF and curve fitting using the Nakagami parameter for the white noise envelope obtained in the simulations.

white noise and Nakagami fitting are shown in Figure 4. These simulation results agree well with those from experiments, in that the average Nakagami parameter and the correlation coefficient respectively are equal to 1.00 ± 0.01 and 0.89, demonstrating that the statistics of white noise envelope obeys the Rayleigh distribution.

Typical ultrasonic backscattered signals from simulations and corresponding Nakagami parameters for phantoms of different scatterer concentrations ranged from 2 to 32 scatterers/mm^2 in the absence of noise interference are shown in Figure 5. The average Nakagami parameter increases from 0.45 to 0.94 associated with increasing scatterer concentration, indicating that the PDF of the ultrasonic backscattered envelope varies from pre-Rayleigh to nearly Rayleigh distribution when the scatterer concentration is increased. The sensitivity of the Nakagami parameter relative to variations in the scatterer concentration, as calculated using the average slope of the curve for the Nakagami parameter as a function of scatterer concentration defined by $\Delta m / \Delta C$ (C represents the scatterer concentration), is 0.016/mm^{-2}. It reveals that the Nakagami parameter is effective for distinguishing various scatterer concentrations in a biological tissue.

Figure 6 is the results of computer simulation for both the typical backscattered signals and the Nakagami parameters as a function of scatterer concentration in the SNR range from 0 to 40 dB. Figure 7 shows the simulated results of the Nakagami parameter as a function of SNR for different scatterer concentrations. The data in Figure 7 were fitted to the following equation, $y = y_0 + a(1 - e^{-bx})$ where values for y_0, a, and b obtained from different scatterer concentrations were calculated and given in Table 1. The sensitivity of the Nakagami parameter as a function of SNR is summarized in Figure 8. For the case of 40 dB SNR, the average Nakagami parameter increases from 0.46 to 0.94 associated with the scatterer concentrations from 2 to 32 scatterers/mm^2. This is approximately the same as that for an infinite SNR, as shown in Figure 5. The average Nakagami parameter is from 0.5 to 0.94 corresponding to the increase of scatterer concentrations with the SNR equal to 20 dB. It was found that the change in the PDF of backscattered envelope from pre-Rayleigh to

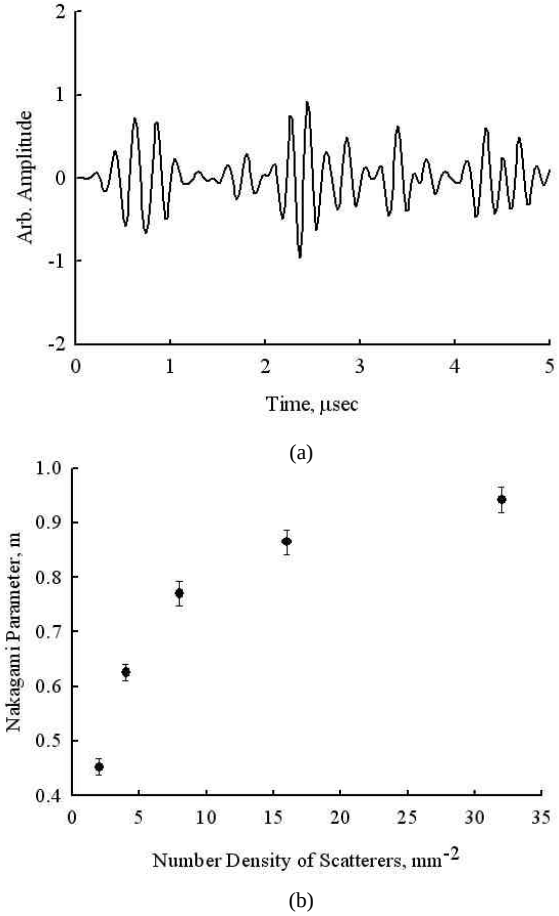


Figure 5. The simulated results without any noise interference. (a) A typical backscattered signal; (b) The Nakagami parameter as a function of scatterer concentration.

Rayleigh distribution is still dependent on the variation in scatterer concentration, and however, the Nakagami parameters at lower scatterer concentrations tend to increase slightly. The sensitivity of the Nakagami parameter is 0.015/mm^{-2}, or about 6% lower than that with an infinite SNR. Moreover, the average Nakagami parameter increases from 0.69 to 0.96 at a SNR of 10 dB. Although the difference between these scatterer concentrations is still discernable by the Nakagami parameter, the decrease of parameter sensitivity from 0.016/mm^{-2} to 0.009/mm^{-2}, approximately 43%, makes this more difficult. The average Nakagami parameter only varies from 0.87 to 0.96 at a SNR of 5 dB (a 81% decrease in sensitivity, from 0.016/mm^{-2} to 0.003/mm^{-2}), resulting in a poor capability of applying the Nakagami parameter to characterize the properties of biological tissues at this SNR. For a SNR of 0 dB, the Nakagami parameters for all scatterer concentrations are close to 1, indicating that the backscattered statistics from different scatterer concentrations are all nearly Rayleigh distributed. For this circumstance, the Nakagami parameter is useless for the ultrasound tissue characterization.

Discussion and conclusions

The Nakagami parameter of ultrasonic backscattered envelope is feasible to be applied for characterizing the

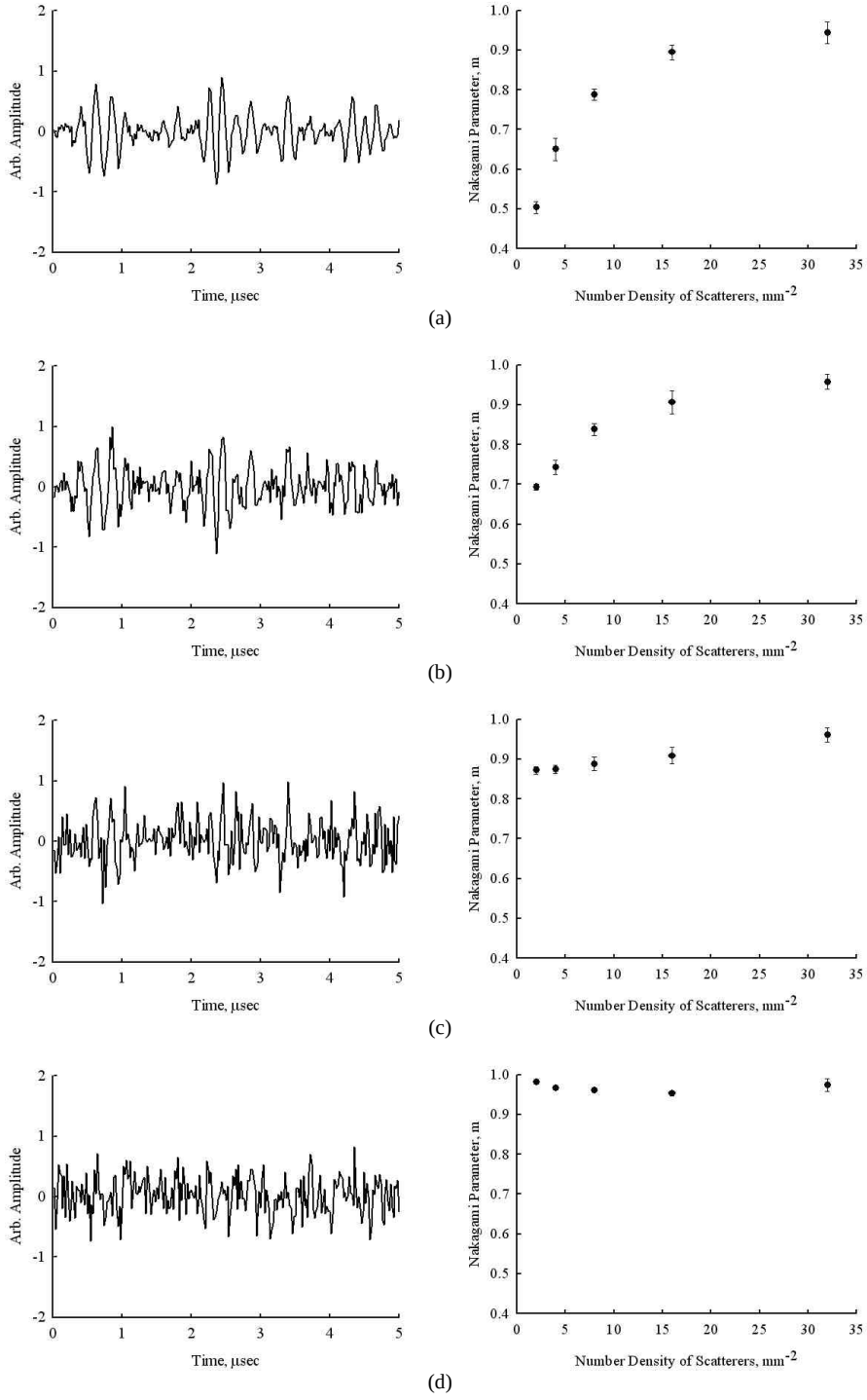


Figure 6. The simulated results for both typical backscattered signals (left) and the Nakagami parameter as a function of scatterer concentration (right) for the following SNR values: (a) 40 dB; (b) 20 dB; (c) 10 dB; (d) 5 dB; and (e) 0 dB.

properties of tissues in clinical diagnoses. Additional information related to the internal structures and the early detection of certain diseases may be provided from the quantitative analysis of ultrasonic backscattered signals, since the statistical parameter calculated using weak ultrasonic backscattered signals is dependent on the shape, size, concentration, density, and other elastic properties of scatterers [13-18]. To better capable of quantifying the properties of tissues, some approaches by improving the algorithm and the measurement system have been also proposed to enhance the sensitivity of the Nakagami parameter for differentiating the variation of scatterer concentration [5,19,20]. However, the performance of the estimated Nakagami parameter is still influenced by noises induced from the environment and instruments. It means that an acceptable SNR for the received ultrasonic signals is a prerequisite for practical applications employing the Nakagami parameter.

In this study, results obtained from both experiments and

Table 1. Values of non-linear regression parameters for the Nakagami parameter in Figure 7 as a function of SNR with the equation of $y = y_0 + a(1 - e^{-bx})$, where r is the correlation coefficient.

scatterer concentration, mm^{-2} / value of parameter	2	4	8	16	32
y_0	1.0048	0.9786	0.9624	0.9506	0.9734
a	-0.5983	-0.3629	-0.1858	-0.0681	-0.0276
b	0.0726	0.0949	0.1115	0.1313	0.1204
r	0.99	0.99	0.99	0.95	0.96

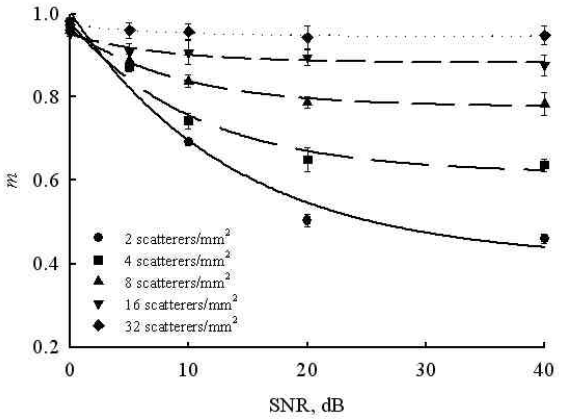


Figure 7. The Nakagami parameter as a function of SNR for different scatterer concentrations.

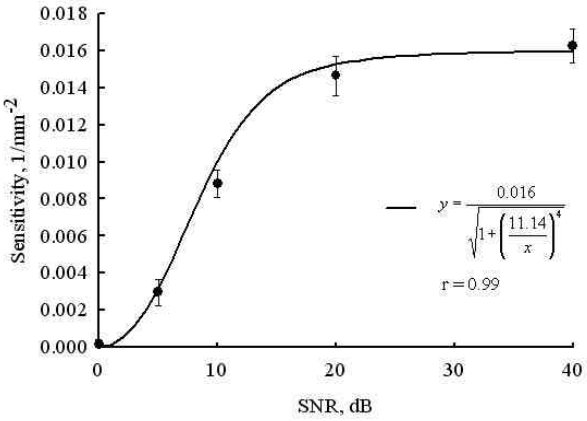


Figure 8. The sensitivity of the Nakagami parameter as a function of SNR.

simulations have demonstrated that white noise could significantly affect the estimation of the Nakagami parameter. Under the free of noise environment, the Nakagami parameter of backscattered signals is able to sensitively differentiate different scatterer concentrations in a tissue, as verified by Figure 5. Nevertheless, the noise is normally unavoidable in a practical measurement. Applying the same measurement system and procedure to a tissue, the estimated Nakagami parameter could be differently associated with the existence of different levels of background noise. Consequently, in addition to influence by the algorithm for estimation and the characteristics of measurement system, the SNR of backscattered signals is showed to be also a key factor for the accuracy and sensitivity of the Nakagami parameter. Owing to white noise typically behaves as a random variable with Gaussian distribution of zero mean, the PDF of its envelope will follow the Rayleigh statistics, as verified in Figures 2 and 4. Accordingly, the coupling of larger background noise with the ultrasonic backscattered echoes tends to modulate the total estimated PDF toward the Rayleigh distribution, which moreover moves the Nakagami parameter toward unity. This is evident from a comparison of the results in Figures 6 and 7. The summarized results in Figure 7 also indicate that the estimation of the Nakagami parameter for various scatterer concentrations reaches the steady state for SNRs above 20 dB, whereas the parameter estimation is SNR-dependent below 20 dB. This affects the sensitivity of the Nakagami parameter in detecting the variation in scatterer concentration, as demonstrated by Figure 8. SNRs of 20, 10, and 5 dB decrease the sensitivity of the Nakagami parameter by 6%, 43%, and 81%, respectively. Moreover, the Figure 8 also depicts that the sensitivity of the Nakagami parameter to the SNR response resembles a high-pass network. Fitting the relationship using a second-order Butterworth high-pass filter shows that a stable sensitivity can be achieved for SNRs above 20 dB. The –3 dB corner SNR is about 11 dB, below which the sensitivity decreases rapidly at an average rate of 10^{-3}/mm^{-2}dB^{-1}. To obtain an adequate sensitivity of the Nakagami parameter, the noise reduction and SNR enhancement are required in the measurement of backscattered signals. For instance, signal processing can be applied to remove partial noise contained in the signals [21]. The use of a focused transducer can enhance the SNR of backscattered signals [17]. A measurement system with anti-noise ability is also useful in isolating external noises [22]. In summary, the estimation of the Nakagami parameter is very sensitive to white noise, which influences the accuracy and sensitivity of using the Nakagami parameter in quantitating the properties of tissues. The present study has demonstrated that ultrasonic backscattered signals with a SNR of at least 11 dB are necessary to allow ultrasound tissue characterization by the Nakagami parameter in clinical diagnoses.

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